

Skyrmion Burst and Multiple Quantum Walk in Thin Ferromagnetic Films

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A giant Skyrmion collapses to a singular point by emitting spin waves in a thin ferromagnetic film, when external magnetic field is increased beyond the critical one. The remnant is a single-spin flipped (SSF) point. The SSF point has a quantum diffusion dynamics governed by the Heisenberg model. We determine its time evolution and show the diffusion process is a continuous-time quantum walk. We also analyze an interference of two SSF points after two Skyrmion bursts. Quantum walks for $S = 1/2$ and 1 are exact solvable. The system presents a new type of quantum walk for $S > 1/2$, where a SSF point breaks into $2S$ quantum walkers. It is interesting that we can create quantum walkers experimentally at any points in a magnetic thin film, first by creating Skyrmions sequentially and then by letting them collapse simultaneously.

Skyrmions are solitons in a nonlinear field theory, playing essential roles in almost all branches of physics[1]. In particular, magnetic thin films have recently attracted much attention owing to real-space observations of Skyrmions[2]. A Skyrmion crystal[3] as well as a single Skyrmion[2] have been identified in chiral magnetic thin films. In spite of their stability guaranteed topologically, an intriguing feasibility arises that we are able to create them and destroy them experimentally by breaking the continuity of the field. This is indeed the case for giant Skyrmions in magnetic thin films[4]. By applying femtosecond optical pulse irradiation focused on a micrometer spot, it is possible to destroy the magnetic order locally[5]. Then, the dipole-dipole interaction (DDI) generates an effective magnetic field, leading to a new magnetic order, that is a Skyrmion spin texture[4]. On the other hand, as the magnetic field increases beyond the critical one, the Skyrmion radius decreases and suddenly shrinks to zero by emitting spin waves. This is the Skyrmion burst.

In this paper we investigate the fate of a Skyrmion after its burst. The notion of the topological stability is lost when the radius becomes in the order of the lattice constant. We consider the two-dimensional ferromagnet with up spins. The spin at the Skyrmion center is precisely oriented downward, which is topologically protected. The remnant of a Skyrmion burst is expected to be a single-spin flipped (SSF) point, which is a single down spin in the up-spin ferromagnet. It has a quantum diffusion dynamics governed by the Heisenberg Hamiltonian. When the spin is $S = \frac{1}{2}$, solving this problem exactly, we find that the down spin hops about two-dimensional lattice points without canting. Namely, a SSF point can be regarded as a two-dimensional continuous-time quantum walker. We have verified manifestations of a quantum walk which differentiate from a classical random walk: The probability density takes the maxima at moving fronts and there appear oscillations between moving fronts. We then analyze an interference of two SSF points after two Skyrmion bursts. The diffusion dynamics is exactly solvable also for $S = 1$. When the spin is higher ($S > \frac{1}{2}$), a SSF point breaks into $2S$ quantum walkers. This is a new type of quantum walk, which we may call a multiple quantum walk. We are able to create quantum walkers experimentally at any points in a magnetic thin film, first

by creating Skyrmions sequentially and then by letting them collapse simultaneously.

A quantum walk is a quantum analogue of a classical random walk[6, 7]. The quantum walk corresponds to the tunnelling of quantum particles into several possible sites, generating large coherent superposition states and allowing massive parallelism in exploring multiple trajectories. The quantum walk is expected to have implications for various fields, for instance, as a primitive for universal quantum computing and systematic quantum algorithm engineering. Recently quantum walks have been experimentally demonstrated using nuclear magnetic resonance[8], trapped ions[9, 10], photons in fibre optics[11] and waveguides[12]. Our work presents an additional example of quantum walks in magnetic thin films.

Giant Skyrmion: We use the classical spin field of unit length, $\mathbf{n} = (n_x, n_y, n_z)$, to describe the spin texture whose scale is much larger than the lattice constant $a = 0.3[\text{nm}]$. The Hamiltonian consists of the anisotropic nonlinear sigma term H_J , the DDI term H_D and the Zeeman term H_Z , whose continuous versions read

$$H_J = \frac{1}{2}\Gamma \int d^2x [(\nabla \mathbf{n}) \cdot (\nabla \mathbf{n}) - \xi^{-2} (n_z)^2], \quad (1)$$

$$H_D = \frac{\Omega}{4\pi} \int d^2x d^2x' \frac{\mathbf{n}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}, \quad (2)$$

$$H_Z = -\Delta_Z \int \frac{d^2x}{a^2} n_z(\mathbf{x}), \quad (3)$$

with the exchange energy Γ , the single-ion anisotropy constant ξ , the DDI strength Ω , and the Zeeman energy Δ_Z . The ground state is the spin-polarized homogeneous state, $\mathbf{n} = (0, 0, 1)$, under the field perpendicular to the plane.

The simplest cylindrical symmetric spin texture is

$$\begin{aligned} n^x(\mathbf{x}) &= -\sqrt{1 - \sigma^2(r)} \cos \theta, \\ n^y(\mathbf{x}) &= -\sqrt{1 - \sigma^2(r)} \sin \theta, \quad n^z(\mathbf{x}) = \sigma(r), \end{aligned} \quad (4)$$

with θ the azimuthal angle. The function $\sigma(r)$ should satisfy the boundary conditions $\sigma(r) = -1$ at $r = 0$ to avoid the multivalueness, and $\sigma(r) \rightarrow 1$ as $r \rightarrow \infty$ to approach the ground state. The spin texture (4) has the Pontryagin number

$Q_{\text{sky}} = 1$, and hence it describes a Skyrmion. In particular, there exists a disk-like spin texture (magnetic bubble domain) with a domain wall at $r = R$. By changing variable as $\sigma(r) = \tanh \tau(r)$, the equation of motion without the DDI term reads

$$\frac{d^2 \tau}{dr^2} + \frac{1}{r} \frac{d\tau}{dr} = \left[\left(\frac{d\tau}{dr} \right)^2 - \left(\frac{1}{r^2} + \frac{1}{\xi^2} \right) \right] \tanh \tau. \quad (5)$$

We solve it by setting $|\tanh \tau| = 1$ in the right-hand side,

$$\sigma(r) = \tanh \left[\log \left[\frac{I_1(r/\xi)}{I_1(R/\xi)} \right] \right], \quad (6)$$

with $I_1(x)$ the modified Bessel function. This is almost the exact solution outside the domain wall. Actually it gives also an excellent approximation to the domain wall. The excitation energy is given by [4]

$$E_{\text{sky}}(R) = \frac{4\pi\Gamma R}{\xi} - \Omega \left[R \ln \frac{R}{d_F} - R \right] + \pi \frac{R^2}{a^2} \Delta_Z + 4\pi\Gamma, \quad (7)$$

where the ground-state energy is subtracted, and d_F is the thickness of the film. The Skyrmion radius R is determined by minimizing $E(R)$ with respect to R . Since R is as large as $1\mu\text{m}$ for typical sample parameters, we have called it a giant Skyrmion [4]. The present Skyrmion is stabilized dynamically by the competition among the DDI, the Zeeman effect and the anisotropy of the film. The number of down spins is of the order $(R/a)^2$, which is about 10^7 for $R = 1\mu\text{m}$.

According to a method of collective coordinate, the dynamics of the Skyrmion radius is governed by the time-dependent Ginzburg-Landau equation,

$$\frac{dR}{dt} = -L \frac{dE_{\text{sky}}(R)}{dR} = -L \left(\frac{4\pi\Gamma}{\xi} - \Omega \ln \frac{R}{d_F} + \frac{2\pi\Delta_Z}{a^2} R \right), \quad (8)$$

where L is the Onsager's constant. The Skyrmion is stable when R is a constant. It becomes unstable when the magnetic field increases beyond a certain critical field. The DDI can stabilize the Skyrmion spin texture no longer. In this case, solving (8), we find the Skyrmion radius to change as

$$R(t) = \left[R_0 + \frac{2a^2\Gamma}{\xi\Delta_Z} \right] \exp \left[-\frac{2\pi\Delta_Z L}{a^2} t \right] - \frac{2a^2\Gamma}{\xi\Delta_Z}, \quad (9)$$

where R_0 is the initial radius. It shrinks to zero ($R = 0$) after a finite time interval,

$$t = \frac{a^2}{2\pi\Delta_Z L} \log \left(\frac{\xi\Delta_Z}{2a^2\Gamma} R_0 + 1 \right). \quad (10)$$

It follows from (6) that, as $R \rightarrow 0$, $\sigma(r) = 1$ for $r > 0$ and $\sigma(r) = -1$ for $r = 0$. The spin texture (4) becomes

$$\mathbf{n}(0) = (0, 0, 1), \quad \mathbf{n}(\mathbf{x}) = (1, 0, 0) \quad \text{for } \mathbf{x} \neq 0. \quad (11)$$

The Skyrmion collapses into a singular point, which is a SSF point.

Skyrmion burst: As the Skyrmion collapses, it emits spin waves: About 10^7 down spins are flipped upward, and the Skyrmion excitation energy $E_{\text{sky}}(R_0)$ is carried away, leaving only the energy E_{SSF} of a SSF point.

To describe spin waves we parametrize the spin field as

$$\begin{aligned} n^x(\mathbf{x}) &= \sigma(\mathbf{x}), & n^y(\mathbf{x}) &= \sqrt{1 - \sigma^2(\mathbf{x})} \sin \vartheta(\mathbf{x}), \\ n^z(\mathbf{x}) &= \sqrt{1 - \sigma^2(\mathbf{x})} \cos \vartheta(\mathbf{x}), \end{aligned} \quad (12)$$

so that the ground state is reached by setting $\sigma(\mathbf{x}) = \vartheta(\mathbf{x}) = 0$. Note that this parametrization is different from that of the Skyrmion (4). The Lagrangian density is given by

$$\mathcal{L} = -\hbar S \sigma \dot{\vartheta} - \mathcal{H}(\sigma, \vartheta), \quad (13)$$

where S is the spin per atom. The Hamiltonian consists of the nonlinear sigma model H_Z and the Zeeman term H_Z . We may neglect the DDI H_D to discuss spin waves. Substituting the above configuration into the Hamiltonian density and taking the leading order terms in σ and ϑ , we obtain

$$\mathcal{H} = \frac{\Gamma}{2} [(\nabla \sigma)^2 + (\nabla \vartheta)^2] + \left(\frac{1}{\xi^2} + \frac{\Delta_z}{2} \right) (\sigma^2 + \vartheta^2). \quad (14)$$

The Euler-Lagrange equations are easily written down. There exists the outgoing propagating wave solution,

$$\sigma(r, t) = J_0(kr) \sin \omega t - N_0(kr) \cos \omega t, \quad (15a)$$

$$\vartheta(r, t) = J_0(kr) \cos \omega t + N_0(kr) \sin \omega t, \quad (15b)$$

up to an appropriate initial condition, where the dispersion relation reads

$$\hbar \omega = \frac{\Gamma}{S} \mathbf{k}^2 + \frac{1}{S} \left(\frac{2}{\xi^2} + \Delta_z \right).$$

The asymptotic behavior is $\sigma(r, t) = \sqrt{2/\pi k r} \sin(kr - \omega t)$, etc., for $r \gg R$. It carries away spins and energies from a Skyrmion.

Quantum walk: The Skyrmion collapses into a singular point in the ferromagnet. The remnant is a SSF point in a ferromagnet. The Skyrmion can be no longer a classical object but a quantum object.

Our concern is about the dynamics of a SSF point after the skyrmion collapse. We show that the dynamics is described as a two-dimensional continuous-time quantum walk as governed by the anisotropic Heisenberg model.

The Heisenberg model is expressed as

$$H_J = \frac{-J}{2} \sum_{\langle i, j \rangle} (S_i^+ S_j^- + S_i^- S_j^+ + 2S_i^z S_j^z) - D \sum_i (S_i^z)^2, \quad (16)$$

from which the nonlinear sigma model (1) follows together with $\Gamma = (1/2)S^2J$ and $\xi^{-2} = 4D/Ja^2$. The Zeeman term is $H_Z = -(\Delta_z/S) \sum_i S_i^z$. The DDI is irrelevant. Here, the index i runs over the two-dimensional lattice points, $i \in \mathbb{Z}^2$. The ground state $|g\rangle$ is the up-spin polarized state defined by

$$S_i^z |g\rangle = S |g\rangle, \quad S_i^+ |g\rangle = 0. \quad (17)$$

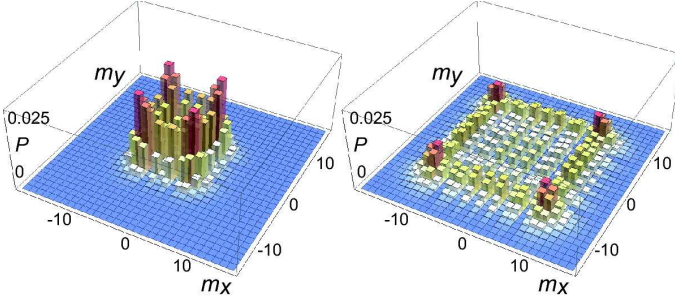


FIG. 1: (Color online) (a) The probability density at $t = 5/J$ and $t = 10/J$. The horizontal axes are m_x and m_y .

A SSF point is generated at $m = (m_x, m_y) \in \mathbb{Z}^2$ by applying $(S_m^-)^{2S}$ to the ferromagnetic ground state,

$$|m\rangle = (S_m^-)^{2S} |g\rangle, \quad (18)$$

where only the spin at the lattice site m is pointed downward.

First we study the case $S = \frac{1}{2}$ in details, since the problem is exactly solvable. It is straightforward to show that

$$\begin{aligned} H|m\rangle &= \frac{-J}{2} \sum_{p=\pm 1} (|m_x + p, m_y\rangle + |m_x, m_y + p\rangle) \\ &+ \left(\frac{J}{2} + \Delta_z\right) |m\rangle. \end{aligned} \quad (19)$$

The energy of a SSF point is $E_{\text{SSF}} = \langle m|H|m\rangle$, which is much smaller than the energy $E_{\text{sky}}(R_0)$ of a Skyrmion.

The dynamics of a SSF point is governed by the Schrödinger equation $i\hbar d\psi/dt = H\psi$. The time evolution reads $|\psi(t)\rangle = e^{-i\frac{t}{\hbar}H} |0\rangle$, where $|0\rangle$ denotes the initial state containing a SSF point at site $m = (0, 0)$. By applying the Hamiltonian to the state containing one SSF point, the point remains at the same site or is shifted to one of the neighboring sites. The resulting state is a coherent superposition of them. It is notable that the spin is strictly parallel to the z -axis: It never cants. The down spin diffuses by time evolution in this way. We can regard a SSF point as a quantum walker.

The quantum diffusion process of a SSF point is governed by a continuous-time quantum walk. Continuous-time quantum walk in one dimension is studied in the refs.[13, 14]. The process is a two-dimensional extension of continuous-time quantum walk. In ref.[13], the probability is obtained by the direct calculation of infinite multiplication of matrix. In ref.[14], Laplace transformation is used. We present a new derivation using the generating function, which is an easy way to generalize to a higher dimension.

Expanding the exponential, we write the time evolution as

$$e^{-i\frac{t}{\hbar}H} |0\rangle \equiv \sum_m \mathfrak{C}_t(m) |m\rangle. \quad (20)$$

Based on a combinatorial method known in the random walk theory, we can show that the coefficient $\mathfrak{C}_t(m)$ is determined

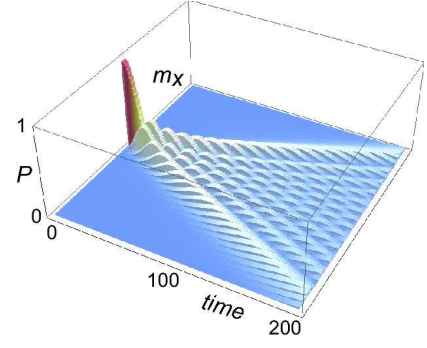


FIG. 2: (Color online) (a) Time evolution of the probability density $P_x(t)$. The horizontal axes are the time t and the site m_x .

from the following generating function,

$$\begin{aligned} &\exp \left[it \left\{ \frac{-J}{2} \left(\xi + \frac{1}{\xi} + \eta + \frac{1}{\eta} \right) + \left(\frac{J}{2} + \Delta_z \right) \right\} \right] \\ &= \sum_{m_x, m_y} \mathfrak{C}_t(m) \xi^{m_x} \eta^{m_y}, \end{aligned} \quad (21)$$

where $m = (m_x, m_y)$. It is determined in a closed form,

$$\mathfrak{C}_t(m) = e^{it(\frac{J}{2} + \Delta_z)} i^{|m_x| + |m_y|} J_{|m_x|}(Jt) J_{|m_y|}(Jt), \quad (22)$$

with the use of the generating function of the Bessel function,

$$\exp \left[\frac{z}{2} \left(s - \frac{1}{s} \right) \right] = \sum_n s^n J_n(z). \quad (23)$$

The probability at site m is given by $P_t(m) = |\mathfrak{C}_t(m)|^2$. The total probability is conserved, $\sum_m P_t(m) = 1$, as is easily checked based on the formula $\sum_n J_n^2(z) = 1$.

We have thus solved the diffusion problem analytically. We show the probability density as a function of site m in Fig.1. The probability density takes the maximum value not at the center but at the fronts. Inside the maximum values, the probability density exhibits an oscillatory behavior in the scale of lattice constant. They are characteristic behavior of a quantum walk. This is highly contrasted to that of a classical random walk, where the probability density is Gaussian with the maximum value taking at the center and not oscillating.

The probability density is factorized as a direct product of the probabilities along the x -axis and the y -axis, $P_t(m_x, m_y) = P_t(m_x)P_t(m_y)$, with

$$P_t(m_x) = |J_{|m_x|}(Jt)|^2. \quad (24)$$

We illustrate the time evolution of $P_t(m_x)$ in Fig.2. Long after the Skyrmion burst, it behaves asymptotically as

$$P_t(m_x) \simeq \frac{2}{\pi Jt} \cos^2 \left[Jt - \frac{2|x|+1}{4} \pi \right]. \quad (25)$$

The velocity of the front propagation is given by $v = 2J/\pi$.

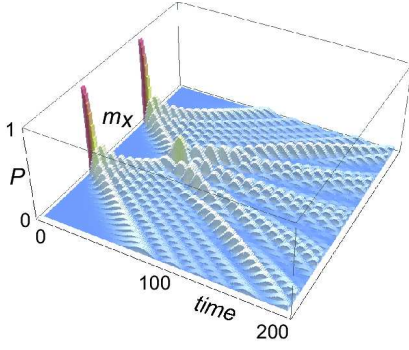


FIG. 3: (Color online) (a) Time evolution of the probability density $P_x(t)$. The horizontal axes are the time t and the site m_x .

We study interference effects of two Skyrmions bursts. We generate two Skyrmions in a thin ferromagnetic film. By applying large magnetic field, two Skyrmions collapse simultaneously and two SSF points are generated. They are quantum mechanical objects, and the probability amplitude is given by a linear superposition of each SSF points. The time evolution of these two SSF points are shown in Fig.3. An interference can be seen between two SSF points. This is a manifestation of quantum mechanical properties of a quantum walk.

Multiple quantum walk: We discuss how the scheme is generalized to the system where the spin is higher than $S = \frac{1}{2}$. In so doing, it is adequate to recapitulate the diffusion process for $S = \frac{1}{2}$ from a slightly different view point. For $S = \frac{1}{2}$, a SSF point at site m is described by $|m\rangle = S_m^-|g\rangle$. When the Hamiltonian (16) acts on this state, the operator S_m^- remains at the same site or is shifted to one of the neighboring sites, as implied by (19). Namely, the operator S_m^- itself can be regarded as a quantum walker.

When the spin is S , by generalizing the above picture, there are $2S$ quantum walkers at the initial SSF point, and then they diffuse. The state is described by

$$|m^1, m^2, \dots, m^{2S}\rangle = S_{m^1}^- S_{m^2}^- \dots S_{m^{2S}}^- |g\rangle, \quad (26)$$

implying that a quantum walker is present at sites m^i . The number of S_m^- is independent of time due to the conservation of the total S^z . Indeed, acting the Hamiltonian H to the state (26), we find a coherent superposition of these states. The initial SSF point is given by (26) with $m^i = (0, 0)$ for all i .

When $S = 1$, there are two quantum walkers. It is easy to calculate $H|m^1, m^2\rangle$ explicitly. The time evolution is also calculable. Since the Clebsch-Gordan coefficient is constant, we obtain a concise formula,

$$\mathfrak{C}_t(m^1, m^2) = \exp[it(J - \Delta_z - D\delta_{m^1, m^2})] \mathfrak{C}_t(m^1) \mathfrak{C}_t(m^2). \quad (27)$$

The probability density is factorizable, $P_t(m^1, m^2) = P_t(m^1)P_t(m^2)$, with (24).

However, since the Clebsch-Gordan coefficient is not constant for $S \geq \frac{3}{2}$,

$$S_m^- |q\rangle = \sqrt{S/2(S/2 + 1) - q(q + 1)} |q - 1\rangle, \quad (28)$$

the analysis is not simple. Each walker does not diffuse independently but interacts each other. Nevertheless, to get a rough picture, we dare to approximate $(S_m^-)^q |q\rangle = C|q - 1\rangle$ with a certain constant C . Then, we find

$$P_t(m^1, m^2, \dots, m^{2S}) \cong \prod_{q=1}^{2S} P_t(m^q). \quad (29)$$

We may regard the diffusion process as a diffusion of $2S$ independent walkers in this approximation.

We have studied the diffusion process at zero temperature. One might wonder if it would survive spin-wave excitations. There exist no problem because spin-wave excitations have a large gap induced by the anisotropy and the external magnetic field.

We have obtained an analytical solution of the time evolution dynamics of the SSF point generated by a Skyrmion burst. Its quantum diffusion process is a spreading oscillatory propagating wave and shows an interference. This is highly contrasted compared to a classical diffusion process, where the dynamics is described by a Gaussian. Our system provides a new approach to investigate properties of quantum walk.

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